Microeconomics Theory Summary

Kevin Low

Spring 2014

Contents

1 Mathematical Fundamentals

1.1 Unconstrained Optimization

Letting $z = F(x, y)$. We assume that z is continuous and twice differentiable; so that the standard techniques of calculus can be applied for optimization

1. First Order Conditions
   (a) $\frac{\partial z}{\partial x} = F_x = 0$
   (b) $\frac{\partial z}{\partial y} = F_y = 0$
2. Second Order Conditions
   (a) If F is strictly concave, then the FOCs are sufficient to guarantee maxima
   (b) Maxima: $\frac{\partial^2 z}{\partial x^2} = F_{xx} < 0$; $\frac{\partial^2 z}{\partial y^2} = F_{yy} < 0$ & $F_{xx}F_{yy} > F_{xy}^2$
   (c) If F is strictly convex, then the FOCs are sufficient to guarantee minima
   (d) Minima: $\frac{\partial^2 z}{\partial x^2} = F_{xx} > 0$; $\frac{\partial^2 z}{\partial y^2} = F_{yy} > 0$ & $F_{xx}F_{yy} > F_{xy}^2$

1.2 Constrained Optimization

1. Formulate Problem: Max (Min) $F(x, y)$ subject to $I = G(x, y)$
2. Form Lagrangian: $L = F(x, y) + \lambda[I - G(x, y)]$
3. Obtain FOCs:
   (a) $\frac{\partial L}{\partial x} = F_x - \lambda G_x = 0$
   (b) $\frac{\partial L}{\partial y} = F_y - \lambda G_y = 0$
   (c) I - G(x,y) = 0
4. SOCs for Lagrangian: Bordered Hessian Tests or Related Concavity/Convexity Conditions (both for objective function and constraint).

   Typically, any pseudoconcave and quasiconvex constrain would suffice to guarantee maximization

1.3 Envelope Theorem

1. Starting with an optimization problem of $\max_{x,y}\ z = F(x, y|w)$ where x,y are variables and w is a parameter
2. Let the solutions to the optimizing problem be $x^* = x(w)$ and $y^* = y(w)$
3. $\frac{dz}{dw} = F_x\frac{dx}{dw} + F_y\frac{dy}{dw} + F_w$
4. Since $F_x = F_y = 0$ at x^* and y^*, $\frac{dz}{dw} = F_w$ (direct equals partial derivative at the optimal)

1.4 Degree of Homogeneity & Euler's Theorem

1. Function homogenous of degree k when $F(tx, ty) = t^k F(x, y)$
2. Differentiating the above with respect to t, $F_x x + F_y y = kt^{k-1}F(x, y)$
3. Therefore $k = 1 \Rightarrow F_x x + F_y y = F(x, y)$

2 Demand

2.1 Firms' Problem

1. Production Function: $Q = F(K, L)$
   (a) Positive Marginal Products: $F_L, F_K > 0$
   (b) Diminishing Marginal Productivity: $F_{LL}, F_{KK} < 0$
   (c) Complementary Factors: $F_{LK} = F_{KL} > 0$
   (d) Above two properties is guaranteed by strict convexity
   (e) Setting $dQ = 0$, the slope of the isoquant is given by $\frac{F_L}{F_K} = -\frac{dK}{dL} = MRTS$ (yes, it is inverted)
   (f) Elasticity of Substitution (Proportional change in $\frac{K}{L}$ relative to changes in $\frac{F_L}{F_K}$:
   $\sigma = \frac{d(K/L)}{d(F_L/F_K)} \frac{(F_L/F_K)}{(K/L)} = \frac{\ln(K/L)}{\ln(F_L/F_K)}$
   (g) Constant Elasticity of Substitution general form: $Q = [K^\rho + L^\rho]^{\frac{\gamma}{\rho}}$ where $\sigma = \frac{1}{1-\rho}$ or $\rho = 1 - \frac{1}{\sigma}$
      i. $\gamma > 1$ implies IRS; $\gamma < 1$ implies DRS
      ii. Cobb-Douglas Function is where $\sigma = 1$; $Q = AK^\alpha L^\beta$; $\alpha + \beta > 1 \Rightarrow$ IRS & $\alpha + \beta < 1 \Rightarrow$ DRS
      iii. If $\sigma = \infty$, function reduces to linear $Q = \alpha K + \beta L$; implying perfectly substitutable inputs which is always CRS
      iv. If $\sigma = 0$, then we have a fixed proportion Leontief Function, $Q = min\alpha K$, which is always CRS
2. Cost Constraint: $C = \nu K + wL$
   (a) w is the wage equilibrium in the labour market in a per-period basis
   (b) ν is the rental rate that is implied based on the market equilibrium of the capital good such that if the equilibrium price of the good is P_K, we know that it has the relationship with rental rate $P_K = \int_0^\infty \nu e^{(-r-\delta)t} dt \Rightarrow \nu = (r + \delta) * P_K$; note that δ is the depreciation rate and r is the single discount rate of the unit of exchange.
3. Derivation of Cost Curve: Minimizing cost with respect to L & K given Q
   (a) Lagrangian: $\mathcal{L} = wL + \nu K + \lambda[Q^* - F(K, L)]$
   (b) FOCs yield:
      i. $w = \lambda F_L$
      ii. $\nu = \lambda F_K$
      iii. $Q^* = F(K, L)$
   (c) Note the above implies $\frac{w}{\nu} = \frac{F_L}{F_K}$ which means the MRTS must coincide with the tangent at Q^*
   (d) Lagrangian multiplier $\lambda = \frac{w}{F_L} = \frac{\nu}{F_K}$ reflects the cost of increasing output at the margin
   (e) Demand for each input can be obtained from combining the first 2 FOCs to relate K in terms of L (ie. $K^* = K(L)$) and then substitute into $Q^* = F(K^*, L^*)$ to obtain $Q^* = F(w, \nu, L^*) \Rightarrow L^* = L(w, \nu, Q^*)$ Same process used to obtain $K^* = K(w, \nu, Q^*)$
   (f) Combining the input demands gives the firm's cost function which is essentially a value function of the firm's optimal input combinations: $C(w, \nu, Q^*) = wL^* + \nu K^*$
   (g) Marginal Cost: $MC = \frac{\partial C}{\partial Q}(w, \nu, Q)$
   (h) Average Cost: $AC = \frac{1}{Q}C(w, \nu, Q)$
4. Input Demand from Shepard's Lemma

(a) Applying Envelope Theorem, we see that the contingent demand for inputs can be obtained from the cost function by differentiating the cost function with respect to the factor price

(b) $\frac{\partial \mathcal{L}}{\partial w} = L^*(w, \nu, Q^*) = \frac{\partial C}{\partial w}$

(c) $\frac{\partial \mathcal{L}}{\partial \nu} = K^*(w, \nu, Q^*) = \frac{\partial C}{\partial \nu}$

2.2 Consumers' Problem

1. Utility Function: $U = U(x, y)$
   (a) Standard Utility Assumptions:
      i. Completeness: Order relationship between 2 bundles are always defined ($\succeq$, $\preceq$, $\prec$, $\succ$, or $\approx$)
      ii. Transitivity
      iii. Continuity
      iv. Ordinal in nature (ranked/ordered but magnitudes don't convey anything else)

   (b) Indifference curve maps out all the bundles that gives the same level of utility

   (c) Marginal Rate of Substitution: MRS $= \frac{U_x}{U_y} = -\frac{dy}{dx}$

   (d) Common Utility Functions
      i. Linear (Perfect Substitutes): $U = \alpha x + \beta y$
      ii. Leontief (Perfect Complements): $U = min\{\alpha x, \beta y\}$
      iii. Cobb-Douglas (In-between): $U = x^\alpha y^\beta$
      iv. General CES: $U = \frac{1}{\sigma}(x^\sigma + y^\sigma)$

2. Consumer Optimization (Utility Maximization)
   (a) Maximization of Utility subject to budget constraint, $I = p_x x + p_y y$

   (b) Standard optimization problem:
      i. Form Lagrangian $\mathcal{L} = U(x, y) + \lambda[I - p_x x - p_y y]$
      ii. First Order Conditions:
      $U_x = \lambda p_x$
      $U_y = \lambda p_y$
      $I = p_x x + p_y y$
      iii. Yields relationship between Price Ratio and MRS: $\frac{U_x}{U_y} = \frac{p_x}{p_y}$
      iv. Individual good's Marshallian Demands, $x^* = D^x(p_x, p_y, I)$ can be obtained by substituting the budget constraint into the FOCs
      v. Combining the resulting demands gives the *Indirect Utility function*, $V(p_x, p_y, I) = U$, which gives the optimal level of utility for any given price vector and income

3. Dual Problem (Expenditure Minimization)
   (a) Minimisation of expenditures for given level of utility, $\upsilon = U(x, y)$

   (b) FOCs will be the same as above but substituting the level of utility gives the Hicksian demands, $x^* = x^h(p_x, p_y, \upsilon)$.

   (c) Combing the Hicksian demands can give the *Expenditure Function*, $E = E(p_x, p_y, \upsilon) = I$

   (d) Expenditure Functions are homogenuous of degree 1, nondecreasing in prices and are concave in prices.

   (e) Because Expenditure Functions and Indirect Utility Functions are inverse of each other; knowing the Expenditure Function (which can be estimated using observables) allows us to work out the Indirect Utility functions which can then be used to model utility

2.3 Primal & Dual Problem

2.4 Slutsky Equations

1. Substitution & Income Effects

   (a) The Hicksian demand curve keeps utility level constant while Marshallian Demand keeps income constant; the gap between them when perturbing the equilibrium gives the income effect since the Hicksian demand accounts fully for the substitution effect

   (b) The Hicksian demand is steeper than the Marshallian demand from the same corresponding price-quantity bundle because Marshallian demand accounts for the additional income effect while the Hicksian demand only reflects the substitution effects (along the same indifference curve; ie. same utility level)

   (c) Slutsky Equations

      i. Own Price Effecs: $\frac{\partial x^*}{\partial p_x} = \frac{dx^h}{dp_x}\Big|_{U=\upsilon} - x^* \frac{\partial x}{\partial I}$

      ii. Cross Price Effects: $\frac{\partial x^*}{\partial p_y} = \frac{dx^h}{dp_y}\Big|_{U=\upsilon} - y^* \frac{\partial x}{\partial I}$

      iii. Note that this is because $E = E(p_x, p_y, \upsilon) = I$ and Shepard's Lemma suggest $x^* = \frac{\partial E}{\partial p_x}$ likewise $y^* = \frac{\partial E}{\partial p_y}$

   (d) We see here then that the changes in quantity demanded as a result of change in prices can be broken down into the substitution effect ($\frac{dx^h}{dp_x}|_{U=\upsilon}$) and the income effect ($x^* \frac{\partial x}{\partial I}$). The substitution effect is always negative (see below) but the income effect can be in either direction.

   (e) Hicksian Law of Demands

      i. Properties of the Hicksian demands are used to analyse the substitution effects; and these properties are essentially the 'laws' for Hicksian Demand

      ii. Negative Own Price Effect [1]: $\frac{dx^h}{dp_x} \leq 0$

      iii. Symmetric Cross-Price Effects [2]: $\frac{dx^h}{dp_y} = \frac{dy^h}{dp_x}$

      iv. Net Substitutes [3]: $\sum_{i \neq j} \varepsilon^h_{i,p_j} \geq 0$

   (f) Marshallian Demand Properties

      i. Homogenous in all prices and income [4]:

2. Compensated & Normal Demands (& Slutsky Demand)

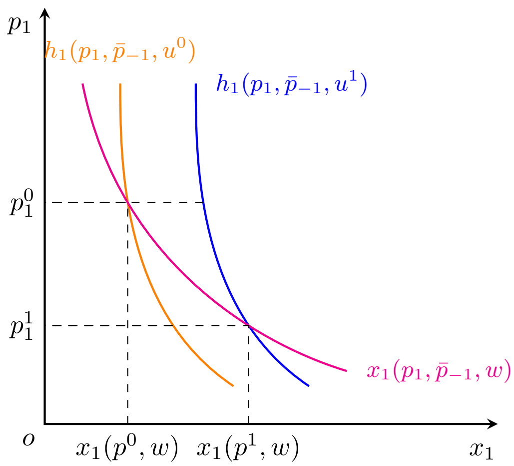
   (a) Compensated Demand refers to the Hicksian Demand and Normal Demand refers to the Marshallian Demand.

[1] By virtue of the fact that the indifference curves generated from the assumptions given are all downward sloping

[2] Proved using Shepard's Lemma combined with Young's Theorem: $\frac{\partial E}{\partial p_x} = x^* = x^h(p_x, p_y, \upsilon) \Rightarrow \frac{\partial^2 E}{\partial p_x \partial p_y} = \frac{\partial x^h}{\partial p_y}$. Thus $\frac{\partial^2 E}{\partial p_x \partial p_y} = \frac{\partial^2 E}{\partial p_y \partial p_x} \Rightarrow \frac{\partial x^h}{\partial p_y} = \frac{\partial y^h}{\partial p_x}$

[3] Proved using the Euler Theorem based on the fact that demand $x^* = x^h(\vec{p}, \upsilon)$ is homogenous in all prices. $\frac{\partial x^h}{\partial p_1} p_1 + \frac{\partial x^h}{\partial p_2} p_2 + ... + \frac{\partial x^h}{\partial p_x} p_x + ... + \frac{\partial x^h}{\partial p_N} p_N = 0$ Since $\frac{\partial x^h}{\partial p_x} \leq 0$, $\sum_{i \neq x} \frac{\partial x^h}{\partial p_i} p_i > 0$. Dividing across by x^* gives $\sum_{i \neq j} \varepsilon^h_{i,p_j} > 0$

[4] Proved using the Euler Theorem based on the fact that $\frac{\partial x_i^*}{\partial p_1} p_1 + \frac{\partial x_i^*}{\partial p_2} p_2 + ... + \frac{\partial x_i^*}{\partial p_N} p_N + \frac{\partial x^*}{\partial I} I = 0$. Dividing across by x_i^* gives the elasticities; more specifically in a 2-goods case: $\varepsilon_{x,p_x} + \varepsilon_{x,p_y} + \varepsilon_{x,I} = 0$



3. Slutsky Elasticities

   (a) Own Price Elasticities: $\varepsilon_{x,p_x} = \varepsilon^h_{x,p_x} - s_x \varepsilon_{x,I}$

   (b) Cross Price Elasticities: $\varepsilon_{x,p_y} = \varepsilon^h_{x,p_y} - s_y \varepsilon_{x,I}$

4. For CES Functions, $\varepsilon^h_{x,p_x} = \sigma(s_x - 1)$, where σ is the elasticity of substitution

3 Supply

3.1 Firms' Supply

1. Supply Curve Derivation

   (a) Deriving the supply will now require the profit maximization based on the firm's cost function as worked out earlier, and the demand functions resulting from the household problem; we now assuming first that firms are price-takers and will only observe price P

   (b) Profit Function: $\pi = P \times Q(L, K) - C(Q(L, K))$

   (c) Optimization involves the FOC: $\frac{\partial \pi}{\partial Q} = \frac{\partial (P \times Q)}{\partial Q} - \frac{\partial C}{\partial Q} = 0$ or $\frac{\partial (P \times Q)}{\partial Q} = \frac{\partial C}{\partial Q}$ or $MR = MC$

   (d) Analysing the Marginal Revenue: $\frac{\partial (P \times Q)}{\partial Q} = P + \frac{\partial P}{\partial Q} Q = P(1 + \frac{Q}{P}\frac{\partial P}{\partial Q}) = P(1 + \frac{1}{\varepsilon_{Q_p}})$ - for a price taking firm, $\varepsilon_{Q_p} = \infty$ so MR = P

   (e) Note that a supply curve for the market is only defined for a perfectly competitive market where firms take prices as given

   (f) Setting P = MC (= MR), we get $\frac{\partial C}{\partial Q}(w, \nu, Q) = P$; inverting this function gives $Q^s = S(w, \nu, P)$

2. Long & Short Run Supply

   (a) The standard supply curve based on the long run marginal cost is the long run supply curve

   (b) The short run supply curve is obtained by holding one or more of the factors fixed: SRMC = $\frac{\partial C}{\partial Q}(w, \nu, Q(\bar{K}, L)) = P \Rightarrow Q = S_{SR}(w, \nu, \bar{K}, P)$

3. Input Demand as function of output price

   (a) Profit Function can be rewritten in further reduced form: $\pi = pF(K, L) - \nu K - wL$ and the FOCs of the optimization are:

      i. $\frac{\partial \pi}{\partial K} = pF_K - \nu = 0$

      ii. $\frac{\partial \pi}{\partial L} = pF_L - w = 0$

      iii. These FOCs implies that the marginal revenue product of each input is equals to their costs at equilibrium

   (b) The FOCs can also be manipulated to generate the input demand as a function of the factor prices and the output price: $K^* = K(\nu, w, p)$ & $L^* = L(\nu, w, p)$

4. Input Demand's Substitution & Output Effect

   (a) Substitution Effect of the Input Demand refers to the input changes as a result of the change in ratio of the cost of inputs given constant level of output.

   (b) Output Effect of the Input Demand refers to the extent of the input changes due to the shift in optimal amount of output as a consequence of the changes in marginal costs induced by the change in ratio of the cost of inputs.

4 Partial Equilibrium

4.1 Market Demand & Supply

1. Aggregate demand in the market is represented by simply aggregating individual demand curves horizontally (summing up the quantities)

2. Aggregate industry supply similarly is generated by aggregating individual firms' marginal cost curves assuming that there are no production externalities.

4.2 Comparative Statics in Elasticities

1. At equilibrium, Quantity demanded equals to the quantity supplied - $D(p,a) = Q^d = Q^s = S(p,b)$

2. Demand-side Shocks

   (a) We assume that the demand shifts due to some exogenous factor change (ie. 'a' shifts)

   (b) Impact of 'a' on 'p': $\frac{dp}{da} = \frac{dD/da}{dS/dp - dD/dp}$

   (c) In elasiticity terms: $\varepsilon_{p,a} = \frac{\varepsilon_{D,a}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$

3. Supply-side Shocks

   (a) We assume that the supply shifts due to some exogenous factor change (ie. 'b' shifts)

   (b) Impact of 'b' on 'p': $\frac{dp}{db} = \frac{dS/db}{dD/dp - dS/dp}$

   (c) In elasiticity terms: $\varepsilon_{p,b} = \frac{\varepsilon_{S,b}}{\varepsilon_{D,p} - \varepsilon_{S,p}}$

4.3 Adjustment Dynamics & Long or Short Run supply

1. Long Run Equilibrium can take time to attain and there are long run reactions on both the demand and supply side.

2. In long run, there is the assumption that the supply is perfectly elastic due to the identical technologies that firms use and it should converge at the industry average cost

3. In short run, supply is virtually fixed but in medium run, retooling of production may allow for limited elasticity of supply (slightly upward sloping)

4. Within relevant range, firms can reorganize supply in long run and that explains the absence of 'rents' and the perfectly elastic supply.

5. If there are production externalities such as some sort of input scarcity then it is possible that long run supply expansion can drive up prices then the long run supply curve would be upward sloping gently; on the other hand, it could be downward sloping if the input gains from economies of scale.

4.4 Economic Rents

1. Economic rents (Profits above what is required to cover opportunity cost) will be competed away in a competitive market

2. Rent emerges in the industry whose output is scarce and cannot be expanded in short run.

3. Case of the Computer Industry

(a) People demand for computing services and the factors to produce that are hardware,application software, operational software and communication technology

(b) In 1960s-70s, IBM had lower cost in hardware and earned the bulk of rents from the production and use of hardware

(c) Dell, Apple, Sun & DEC got drawn into the market and the immense profits started declining as the price of computing services fell

(d) As hardware became less of a limiting factor, quality of computing services became limited by software so rents became associated with Microsoft

(e) With the rise of the Internet the supply bottleneck turned towards communications technology and that marked the rise of firms like Cisco. Unfortunately, their expected rise in demand didn't quite pan out as well and their huge investments to gain network economies drove them into some difficulties.

(f) Consulting, outsourcing and computing management appears to be the coming limiting factor for businesses' use of technology.

4.5 Elasticity Analysis

1. In this section, we prove the results of the comparative Statics in Elasticities that was presented in part 4.2.

2. Given the elasticities, we can examine the relationship between an exogenous factor to demand or supply with the equilibrium prices. Such analysis of elasticity allows us to estimate the price elasticities of these factors.

3. Suppose we have Demand $Q^d = D(p, y)$ and Supply $Q^s = S(p, w)$ where we can think of y as income and w as the weather.

4. Given the price elasticity of demand ($\varepsilon_p^d = \frac{dQ^d}{dp}\frac{p}{Q^d}$) and supply ($\varepsilon_p^s = \frac{dQ^s}{dp}\frac{p}{Q^s}$) as well as the income elasticity of demand ($\varepsilon_y^d = \frac{dQ^d}{dy}\frac{y}{Q^d}$) or the weather elasticity of supply ($\varepsilon_w^s = \frac{dQ^s}{dw}\frac{w}{Q^s}$), we are to work out the price elasticity of income ($\varepsilon_p^y = \frac{dy}{dp}\frac{p}{y}$) or weather ($\varepsilon_p^w = \frac{dw}{dp}\frac{p}{w}$).

5. Suppose we simply want to examine the impact of changes in income on the changes in equilibrium price. We know that at equilibrium, $Q^d = Q^s = D(p, y) = S(p, w)$ and we apply total differentiation: $dQ^d = D_p dp + D_y dy$ and $dQ^s = S_p dp + S_w dw$

6. Equating the above, $S_p dp + S_w dw = D_p dp + D_y dy$; and since $dw = 0$, we get $S_p dp = D_p dp + D_y dy \Rightarrow dp(S_p - D_p) = D_y dy \Rightarrow \frac{dp}{dy} = \frac{D_y}{S_p - D_p} \Rightarrow \frac{dp}{dy}\frac{y}{p} = \frac{D_y}{S_p - D_p}\frac{y}{p} \Rightarrow \varepsilon_y^p = \frac{dQ}{dy}\frac{y}{Q}[\frac{p}{Q}(\frac{dQ^s}{dp} - \frac{dQ^d}{dp})]^{-1} \Rightarrow \varepsilon_y^p = \frac{\varepsilon_y^d}{\varepsilon_p^s - \varepsilon_p^d}$

5 General Equilibrium

5.1 Core & its Existence

1. Concept of Core: A set of feasible unblocked allocations

2. Blocking occurs when there is an alternative means of allocation within different coalitions that are Pareto-improving (improving some welfare without making others worse off)

5.2 Theory of Prices

1. Concept of Excess Demand: $Z_k(P) = \sum_i x_k^i(P, Pe^i) - \sum_i e_k^i$

   (a) P is the vector of all prices

   (b) x_k^i is the demand for the k^{th} good by the i^{th} consumer

   (c) e_k^i is the endowment of good k for consumer i

   (d) $Z_k(P)$ is the excess demand for good k

   (e) Pe^i is essentially the 'income' of the consumer i

2. Walras' Law: Sum of Excess Demand equals 0 because consumers spend all their endowment $\Rightarrow PZ(P) = 0$

   (a) Budget Constraint of consumers hold with equality: $\sum_k^n P_k(x_k^i(P, Pe^i) - e_k^i) = 0$

   (b) Summing the Budget Constraints over all the consumers: $\sum_i \sum_k^n P_k(x_k^i(P, Pe^i) - e_k^i) = 0$

   (c) Factoring out prices: $\sum_i P_k(\sum_k^n x_k^i(P, Pe^i) - \sum_k^n e_k^i) = 0$

   (d) Substituting Excess Demand: $\sum_i P_k(Z_k(P)) = 0 \Rightarrow PZ(P) = 0$

3. Does such a set of prices exist such that $Z_k(P) = 0$?

   (a) Using Walras' Law, we see that $\sum_i P_k(Z_k(P)) = 0$, so we know that at equilibrium, $Z_k(P) \begin{cases} = 0 & \text{for } P_k > 0 \\ \geq 0 & \text{for } P_k = 0 \end{cases}$

   (b) Therefore, if not at equilibrium, $Z_k(P) > 0 \Rightarrow \Delta P_k > 0$ and $Z_k(P) < 0 \Rightarrow \Delta P_k < 0$

   (c) We then define a mapping of prices to themselves as a means of examining the way prices change with excess demand: $F_k(P) = \max\{P_k + Z_k, 0\}$

      i. Max condition ensure non-negative prices

      ii. $F_k(P) = P_k + Z_k$ implies that prices moves up or down in accordance to excess demand

      iii. We further normalize prices so they sum to 1: $\sum_k^n F_k(P) = 1$

   (d) Invoking Brouwer's Fixed Point Theorem (that simply mentions the presence of the fixed point without giving a means of finding it): Any continuous mapping $F(x)$ of a closed, bounded (ie. compact) convex set into itself has at least one fixed point where $x^* = F(x^*)$

   (e) Therefore, with $P_k^* = F_k(P)$, we know that this equilibrium exists and would be consistent with Walras' Law.

   (f) Whether we will arrive at the equilibrium is not guaranteed.

4. Walrasian Equilibrium Allocation

   (a) It is an allocation of goods based on initial endowments and equilibrium prices:
   $X(P^*) = (X^1(P^*, P^*e^1), \cdots, X^i(P^*, P^*e^i), \cdots)$

   (b) Each component is the n-vector of goods demanded and received by consumer i at equilibrium price vector P^*.

(c) With a set of endowment e, $W(e)$ provides the set of Walrasian equilibrium allocations

(d) Every WEA is in the core but as the economy enlarge in terms of actors, the core shrinks and the number of points in the core converges towards the potential WEA. In a large market, WEA coincide with the core and the equilibrium allocation becomes unique.

(e) On the PPC with aggregate social welfare function (existence of which is incredibly dubious), we can see that $\frac{MU_y}{MU_x} = \frac{P_y}{P_x} = \frac{MC_y}{MC_x} \Rightarrow$ Technical Costs of production and the trade-off based on preferences are reflected in the relative prices

5. Welfare Theorems

(a) First Welfare Theorem: Every Walrasian Equilibrium Allocation is Pareto Efficient (assuming conditions of full information, zero transaction costs, and zero market power)

(b) Second Welfare Theorem: Any desired distribution can be obtained through competitive markets accompanied by appropriate transfers of initial endowments (just do not mess with prices and marginal incentives).

6 Application of Partial & General Equilibrium Analysis

6.1 Specific Results

6.1.1 Setup of Model

1. 2 Sectors: Corporate (producing X) & Non-Corporate (producing Y)
2. 2 Factors: Capital (K) & Labour (L)
3. Demand Assumption: Cobb-Douglas preferences with equal weights on each good ($S_x = S_y = \frac{1}{2}$)
4. Cobb-Douglas Production Functions are assumed to have equal weights: $Q = K^{\frac{1}{2}}L^{\frac{1}{2}}$
5. Pretax $r = \frac{r^*}{1-T}$ where r^* is the after-tax required rate of capital and T is the rate of corporate tax
6. Numerical Assumptions:
   (a) Total Output = 1200
   (b) $r = w = 1$ in the beginning before the tax

6.1.2 Case of Cobb-Douglas Production in both sectors (taxed & untaxed)

1. With the Cobb-Douglas production function assumed, $S_x = S_y = \frac{1}{2}$ & $P_xX = P_yY = \frac{1}{2} \times 1200 = 600$
2. Since the value of Marginal Products Rule hold such that return on factors would be equals to the marginal product:
   $rK_x = \frac{1}{2}P_xX$ & $rK_y = \frac{1}{2}P_yY$; $wL_x = \frac{1}{2}P_xX$ & $wL_x = \frac{1}{2}P_yY$
3. Since we have made the assumption that $r = w = 1$; $K_x = K_y = 300$ & $L_x = L_y = 300$
4. With a tax of 50% on the corporate sector, the pretax amounts paid to each factor do not change $\Rightarrow$ Tax $+rK_x = 300 \Rightarrow rK_x = 150$
5. In order to equalize post-tax return on capital, the marginal product of capital in X must be twice that of Y $\Rightarrow \frac{K_x}{K_y} = 2$ and since $K_x + K_y = 600$, $K_x = 200$
6. With a tax rate of 50%, $r(200) = 150 \Rightarrow r = 0.75 = \frac{3}{4}$
7. Capital thus bears the **full burden** of the tax

6.1.3 Case of Cobb-Douglas Production Y (untaxed) & Fixed Proportions in X (taxed)

1. With the same initial assumptions, the tax this time will release factors from X since there is no means of substituting labour for capital in the case of Fixed Proportions
2. These factors will move to sector Y where we know that production will increase; and since $P_yY = 600$, we know that P_y will fall
3. Both r and w should fall by the same amount since the shift in both factors is symmetric - therefore both factors bears the burden of the taxes jointly
4. Production in X: $P_xX = 600 =$ Tax $+rK_x + wL_x$, and since Tax $= rK_x$, $P_xX = 2rK_x + wL_x$; and due to Fixed Proportions, $L_x = K_x$
5. Production in Y: $rK_y = wL_y = 300$; and as mentioned earlier, due to equal release of factors, $L_y = K_y$
6. Combining the equations above, we get $300 = r(600 - K_x) = rK_y = w(600 - L_x) = w(600 - K_x) \Rightarrow r = w$

7. We then see that $rK_x = 200$ and $r(600 - K_x) = 300$ so $3rK_x = 2r(600 - K_x) \Rightarrow 5K_x = 1200 \Rightarrow K_x = 240$ & $K_y = 360$

8. Therefore, $rK_y = 300 \Rightarrow r = \frac{5}{6} = w$, which means that both labour and capital bears equal share of the burden - labour is forced to shoulder some of the burden from the capital tax due to the fixed proportion nature of the taxed industry.

6.1.4 Case of Cobb-Douglas Production X (taxed) & Fixed Proportions in Y (untaxed)

1. With the same initial assumptions, this time we see that when there is a tax on K_x, there is a shift of input mix towards labour and the capial shifts to Y; but because Y is Fixed Proportion, it will absorb labour from X over as well. This implies that the production effect dominates the substitution effect.

2. Due to the Cobb-Douglas production in X, we should expect that $wL_x = 300$ nevertheless so the fall in L_x must imply w increases (demand for labour increases in both sectors); the burden on capital is more than the size of the tax so as to 'subsidise' labour in order to restore the efficient input mix.

3. We know that the Fixed Proportions implies $L_y = K_y$ but at the same time the other sector is forced to have that same mix so $L_x = K_x$

4. Hence, given that the actual return r is now half of the pre-tax return on capital, $wL_x = 2rK_x = 2rL_x \Rightarrow w = 2r$; incorporating this into $600 = rK_y + wL_y \Rightarrow 600 = r(600 - K_x) + 2r(600 - L_x)$

5. Using the fact that $300 = wL_x = 2rK_x$, we know ha $4rK_x = 600$ and therefore, combining that and above, we have $4rK_x = r(600-K_x)+2r(600-L_x) \Rightarrow 4K_x = 600-K_x+1200-2K_x \Rightarrow 7K_x = 1800 \Rightarrow K_x = \frac{1800}{7}$

6. Consequently, $L_x = \frac{1800}{7}, L_y = K_y = 600 - \frac{1800}{7}$ and $w = \frac{7}{6} \Rightarrow r = \frac{7}{12}$, which means that labour is earning excess rents and capital is bearing more than the full burden of he tax!

6.2 Generalized Conclusions

1. Only if taxed industry is more labour intensive can labour bear more of the tax (in proportion to its initial share) than capital

2. If the elasticity of substitution ($\sigma_{L,K}$) is greater than the elasticity of demands (of both goods in absolute value terms), then capital bears more of the tax

3. The greater the elasticity of substitution in the untaxed sector ($\sigma^y_{L,K}$), the greater the tendency for both labour and capital to share the burden of the tax (in proportion to their initial income shares)

4. The greater the elasticity of substitution in the taxed sector ($\sigma^x_{L,K}$), the less the impact on the post-tax rate of return

5. $\sigma^x_{L,K} = \sigma^y_{L,K} \Rightarrow$ Capital bears burden

6. $\sigma^x_{L,K} < \sigma^y_{L,K} \Rightarrow$ Capital bears less than full burden

7. $\sigma^x_{L,K} > \sigma^y_{L,K} \Rightarrow$ Capital bears more than full burden

6.3 Application in Today's World

1. Assuming that in today's world, capital is mobile and r is fixed globally while Labour is immobile, we examine the incidence of the taxes.

2. Because of the mobility of capital, the labour is forced to bear the full burden of taxes since the rate of return for capital is fixed globally - it can simply 'escape' if post-tax returns are lower than the global rates.

7 Monopoly & Imperfect Competition

7.1 Monopoly & Monopsony

7.2 Models of Imperfect Competition

7.2.1 Cartel Oligopoly

1. Firms act in concert as if it is a single entity and thus seek to maintain total output at the monopolist level.
2. Profit Function: $\Pi = P(Q)Q - \sum_i^n c_i(q_i)$ where $Q = \sum_i^n q_i$
3. Profit-maximizing requires $\frac{\partial \Pi}{\partial q_i} = P(Q) + Q\frac{\partial P}{\partial q_i} - \frac{\partial c_i}{\partial q_i} = 0 \Rightarrow MR = MC_1(q_1) = \cdots = MC_n(q_n)$
4. If the firms are all identical then they would each produce the same amount
5. Note that if a single country is the sole producer/exporter of a particular good to the world; the government can generate and extract monopoly rents even if the firms are engaged in perfect competition domestically; this is done by setting a tariff to bring the exported prices up to the level of the monopoly price so that output abroad is restricted.

Two Firm Example
Demand: $P = 130 - Q$, where $Q = q_1 + q_2$
Marginal Cost: $\frac{\partial c_1}{\partial q_1} = \frac{\partial c_2}{\partial q_2} = 10$
Sharing Output Identically: $q_1 = q_2 = \frac{Q}{2}$
Profits: $\Pi = (130 - Q)Q - 2TC_i(\frac{Q}{2})$
First Order Conditions: $\frac{\partial \Pi}{\partial Q} = 120 - 2Q = 0 \Rightarrow Q = 60 \Rightarrow q_1, q_2 = 30$
Equilibrium Price: $P^* = 130 - 60 = 70$
Equilibrium Profits: $\Pi^* = (70 - 10) \times 60 = 3600$

7.2.2 Cournot Competition

1. Firms act independently but simultaneously
2. Each firm knows the Best-Response of another firm and give its best response to that
3. Profit Function: $\pi_i = P(Q)q_i - c_i(q_i)$
4. First Order Condition: $\frac{\partial \pi_i}{\partial q_i} = P(Q) + q_i\frac{\partial P}{\partial q_i} - MC_i = 0 \Rightarrow P(Q) + q_i\frac{\partial P}{\partial q_i} = MC_i$
5. Same model as the 'Tragedy of the Commons' problem

Two Firm Example
Demand: $P = 130 - Q$, where $Q = q_1 + q_2$
Marginal Cost: $\frac{\partial c_1}{\partial q_1} = \frac{\partial c_2}{\partial q_2} = 10$
Profits: $\pi_i = (130 - q_i - q_{-i})q_i - TC_i(q_i)$
First Order Conditions: $\frac{\partial \pi_i}{\partial q_i} = 130 - 2q_i - q_{-i} - MC_i = 0 \Rightarrow q_i = \frac{120 - q_{-i}}{2}$
By Symmetry: $q_1^* = q_2^* = 40$ from solving the above equation
Equilibrium Price: $P^* = 130 - 80 = 50$
Equilibrium Profits: $\Pi^* = (50 - 10) \times 80 = 3200$

7.2.3 Dominant Firm, Stackelberg Competition, Conjecture Variation Model

1. We have a single Price Leader who has the first mover advantage but would work out the other firm's reaction function and price according to its residual demand

2. Accounts for his action's impact on others while others' actions are taking his as given.

3. Theoretically, with a competitive fringe, the Price Leader would have lower marginal cost than the rest of the firms.

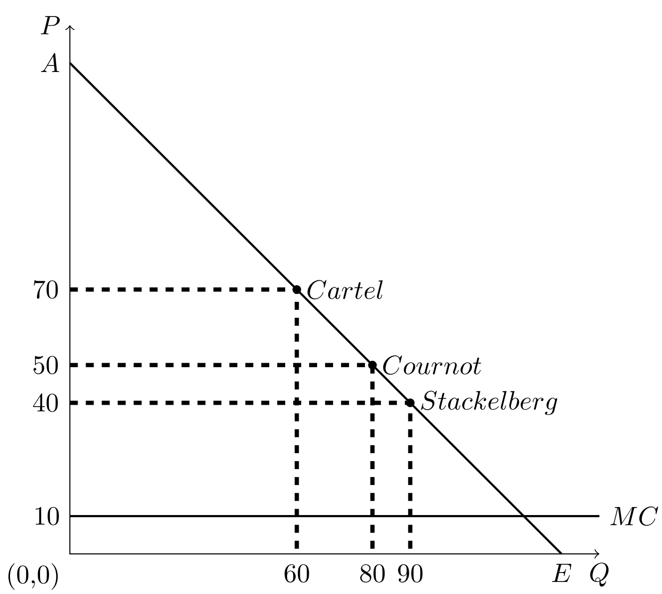
Two Firm Example
Demand: $P = 130 - Q$, where $Q = q_1 + q_2$
Marginal Cost: $\frac{\partial c_1}{\partial q_1} = \frac{\partial c_2}{\partial q_2} = 10$
Follower Profits: $\pi_2 = (130 - q_1 - q_2)q_2 - TC_2(q_2)$
Follower First Order Conditions: $\frac{\partial \pi_i}{\partial q_i} = 130 - 2q_2 - q_1 - MC_i = 0 \Rightarrow q_2 = \frac{120-q_1}{2}$
Leader Profits: $\pi_1 = (130 - q_1 - 60 + \frac{q_1}{2})q_1 - 10q_1$
Leader First Order Condition: $\frac{d\pi_1}{dq_1} = 130 - 2q_1 - 60 + q_1 - 10 = 0 \Rightarrow q_1^* = 60 \Rightarrow q_2^* = 60 - 30 = 30$
Equilibrium Price: $P^* = 130 - 60 - 90 = 40$
Equilibrium Profits: $\Pi^* = (40 - 10) \times 90 = 2700$

7.2.4 Comparison of Models

1. It would appear that given constant marginal cost, the consumer surplus is highest in the case of a dominant firm (Stackelberg).

2. The presence of a price leader trying to dominate the market results in an 'oversupply' from a cartel/collusive perspective and thus improves consumer welfare

Model	Prices	Quantities	Joint Profits
Cartel	70	60	3600
Cournot	50	80	3200
Stackelberg	40	90	2700



8 Price Discrimination & Pricing Issues

1. Price Discrimination is essentially charging different prices to different consumers or for different quantities
2. Conditions for Price Discrimination:
   (a) Firm must have pricing power in the market
   (b) Firm must have some information about willingness of consumers to pay - be it consumer groups/types, specific consumer information or aggregate information
   (c) Firm must be able to prevent resale of goods
3. Actual Price Discrimination Cases:
   (a) Alcoa vertically integrating with downstream industries with highly price-elastic demands so as to charge higher mark-ups to the industries with low price-elasticity of demand.
   (b) Labour Unions contracting with firms to force a certain wage-labour pair contract on industrial plants.
   (c) Senior and Student discounts; for a variety of non-transferable services and goods.
   (d) Coupons requiring time and effort to find, clip and make payments with.
   (e) Agricultural marketing orders; classified pricing schedule where farmers supply fresh produce at a higher price in the primary (fresh foods) market and a lower price in the secondary (exports or processed foods) market.

8.1 First Degree Price Discrimination

1. Two-part Tariff: pricing a P=MC for each unit of good and then charging a fixed-fee equals to the consumer surplus.
2. Volumetric Pricing Schedule: A price schedule with price-quantity pairs.
3. For goods where each consumer buys a single unit of good, bargaining and information gathering through other means is conducted to assess willingness to pay and offer price is made.

8.2 Third Degree Price Discrimination

1. Used in the case where the firm can identify the group a consumer falls into and knows the aggregate level willingness to pay for each group.
2. Dividing up customers into consumer groups with different price elasticity of demand and thus different willingness to pay.
3. Supply a different quantity of good to different markets by charging different prices in each market; equilibrium quantities and prices obtained by setting $MR_1 = MR_2 = MC$ where 1 and 2 are the different markets that the monopoly supplies

8.3 Second Degree Price Discrimination

1. Used when the firm knows there are different groups of consumers with different willingness to pay but the firm is unable to verify which group the consumer falls in; therefore this uses self-selection techniques.
2. Can be conducted with imperfect information; and can be done through quantity or quality restrictions conditional on the pricing.

8.3.1 Simple Two Part Tariff

1. Works by having firms set prices such that there is a combination of a fixed fee and a marginal fee with the fixed fee equals to the consumer surplus of those with the lowest willingness to pay

2. Firm can do even better if it is able to restrict the quantity consumed under different fixed tariff regimes; this generates the Price-Quantity Pairing.

8.3.2 Price-Quantity Paired Tariffs

1. Offering a 'take-it-or-leave-it' pricing where the price and quantities are tied.

2. Requires the indifference curves of different consumer types to exhibit the single-crossing property.

3. Essentially offering the low-demand type a package that would extract all their surplus completely

4. And then a more attractive package with much higher quantity or quality that would give the high-demand type a higher level of utility than if they had picked the package of the low-demand type. By construction, it is impossible that the high demand type spends less on aggregate but they do have more surplus (positive) than the low-type (zero).

8.4 Strategic Bundling

1. Offering a combination of a la carte menu items and also all-you-can-eat buffet prices on food items; essentially a kind of second degree self-selecting mechanism but extends beyond 1 product

2. Fish & Pie or the AER-JEL Problem from Friedman's Price Theory

8.5 Durable Good Monopoly Pricing Problem

1. Question of whether a monopoly should be renting or selling it's goods if it is durable

2. Simple Case (where the monopoly do not care about future profits; discount rate is 1)
   (a) Selling and renting is no different
   (b) Sale price will just be related to the rental price discounted; essentially the firm simply takes the 'stock demand' (discounted demand of the flow of services from the good) and then price accordingly
   (c) Sale Price, $P = \sum_t^\infty \frac{R_t}{(1+r)^t}$ where R_t is the per-period rental rate.

3. Coase Conjecture (where monopoly cares about future profits and consumers rationally expects monopolist to remain in market)
   (a) Reducing pricing power because of inability to commit to leaving the market; consumer expects prices to fall (due to monopolist lowering price to take up residual demand)
   (b) Monopolist forced to compete with future-self as well as secondary market (relating to the ALCOA problem)
   (c) Therefore, renting is the best option since it preserves monopoly power; the excuse/upside is that the monopolist will maintain and can recycle the products

4. Examples of such goods and problems:
   (a) IBM Mainframes, which are extremely expensive anyways but only rented
   (b) Xerox Copiers, only rented initially but later forced to sell

8.5.1 ALCOA Problem - Competitive Fringe

1. Judge Hand concludes that ALCOA is 'guilty' of monopolizing the market because they were dominant in production, and controlled the supply to the secondary market by monopolizing primary production

2. The truth of the case is such that the aluminum market was growing at that time with demand expanding and therefore the competitive fringe recycling aluminum would never be able to supply that growing demand; ALCOA receives the residual demand that would always help to sustain its monopoly power.

3. ALCOA doesn't suffer from the problem of competition from secondary market or its previous production (Coase Conjecture) due to the fact that demand was growing

9 Information Asymmetry & Lemons Problem

9.1 Collapse of Markets Under Information Asymmetry

1. Information problems of moral hazard and adverse selection; in particular, the lemons principle demonstrates the problem of adverse selection
2. Adverse Selection is where there are 'hidden types' (signalling & screening problems)
   (a) Insurance; healthy vs unhealthy types
   (b) Labour Market; education as a means of signalling ability
   (c) Vendor/Product Quality; use of consumer reports
3. Moral Hazard is where there are 'hidden actions' (misalignment of incentives)
   (a) Principal-Agent Problem of any sorts (Tenured Professor, Share-cropper, etc.)
   (b) Insurance Post-sales
   (c) Opportunistic Behaviours in joint-investments

9.2 Mathematical Illustration of Lemon Principle

9.2.1 Setup of Model

1. Quantity demanded $Q^d = D(p, \mu)$
2. Quantity supplied $Q^s = S(p)$
3. Sellers Utility: $U_s = M + \sum_i^n x_i$
4. Buyers Utility: $U_b = M + \sum_i^n \frac{3}{2} x_i$
5. Assume all agents are von Neumann-Morgenstern maximizers of expected utility
6. Assume the sellers has a total of N cars with uniformly distributed quality x, $0 \leq x \leq 2$
7. Assume price of other goods M is unity.

9.2.2 Asymmetric Information

1. Seller's Perspective:
   (a) Because of perfect substitutability, demand is such that all income is devoted to buying the good if quality exceeds price
   (b) $D_s = \begin{cases} \frac{Y_s}{p} & \text{if } \frac{\mu}{p} > 1 \\ 0 & \text{if } \frac{\mu}{p} < 1 \end{cases}$
   (c) $S_s = \begin{cases} \frac{pN}{2} & \text{if } p \leq 2 \\ 0 & \text{otherwise} \end{cases}$
   (d) Average quality $\mu = \frac{p}{2}$
2. Consumer's Perspective (Buyers):
   (a) Similarly, because of the relative substituability of M and x; we have the following demands:

(b) $D_b = \begin{cases} \frac{Y_b}{p} & \text{if } \frac{\mu}{p} > \frac{2}{3} \\ 0 & \text{if } \frac{\mu}{p} < \frac{2}{3} \end{cases}$

(c) $S_b = 0$

3. Aggregate Market Equilibrium

   (a) Aggregate Demand: $D = \begin{cases} \frac{Y_b+Y_s}{p} & \text{if } p < \mu \\ \frac{Y_b}{p} & \text{if } \mu < p < \frac{3\mu}{2} \\ 0 & \text{if } p > \frac{3\mu}{2} \end{cases}$

   (b) Aggregate Supply: $S = \begin{cases} \frac{pN}{2} & \text{if } p \leq 2 \\ 0 & \text{otherwise} \end{cases}$

   (c) At p, the average quality is $\mu = \frac{p}{2}$ which means that there is no equilibrium! No p where $D = S$

9.2.3 Symmetric Information

1. With symmetric information, the supply curve is transformed into a fixed quantity supply at N when prices are above expected quality $\mu = 1$

2. Aggregate supply becomes $S = \begin{cases} N & \text{if } p > 1 \\ 0 & \text{otherwise} \end{cases}$

3. Aggregate Demand would be $D = \begin{cases} \frac{Y_b+Y_s}{p} & \text{if } p < 1 \\ \frac{Y_b}{p} & \text{if } \mu < p < \frac{3}{2} \\ 0 & \text{if } p > \frac{3}{2} \end{cases}$

4. Equilibrium will be different depending on relative value of N, Y_b and Y_s

   (a) $p = 1$ if $Y_b < N$

   (b) $\frac{Y_b}{N}$ if $\frac{2Y_b}{3} < N < Y_b$

   (c) $p = \frac{3}{2}$ if $N < \frac{2Y_b}{3}$

9.3 Insurance Pricing Problem

1. Related to Price discrimination but more of an issue of informational asymmetry

2. When perfect type-screening is possible;

   (a) Individuals will make use of the market opportunities to trade their risks and get themselves on the line of certainty

   (b) For a low-risk individual, the slope to the point on the line of certainty is steeper than the high-risk individual (because the expected state of gain is higher)

3. Without perfect type-screening;

   (a) Pooling equilibrium not viable; the insurance policy that appears viable is one where the cross of the indifference curve intersects exactly with the pooled-insurance profits (dependent on the composition of the types).

      i. However, there are areas above the U_L and below the U_H that enables insurers to 'cream-skim' and cause the market to collapse.

ii. As long as the market allows free entry for insurance package that appeals only to the low risk types, the migration of low-risk to that package eliminates the pooling profits (it stops being economically viable)

(b) Separating Equilibrium possible and viable under certain situation; we can find a U_H that optimizes at the full insurance contract and then offer the low-risk a partial insurance at a point below the U_H but still presents an improvement over the risk-endowment.

i. In this separating equilibrium, the high-risk types fully insure and pays the actuarially fair price. Economic viability is assured in this case.

ii. Low-risk types are made to prefer the partial-insurance at actuarially fair prices because any full insurance package that they prefer would be strictly preferred by the high-risk types as well which makes the separation impossible.

(c) Signalling and screening are means the insurance companies can identify types and price more accurately. In particular, the willingness to pay for different packages in the separating equilibrium is a form of signalling. The low-risk types pay the information rents through partial insurance (they are taking on some risk in order to prove they are low-risk)

10 Externalities & Coase Theorem

10.1 Traditional Pigouvian Externalities

1. Typically the basic assumption is that of perfectly competitive, 'functioning' markets but there is divergence between private and social marginal costs that influences behaviours of agents such that welfare improvements can be made.

2. Positive Externality defined by $PMB < SMB$ while Negative Externality defined by $SMC > PMC$

   (a) Examples of Negative Externalities include noise, different forms of pollution, road congestion

   (b) Examples of Positive Externalities include education, national defence

10.2 Public Good Diversion

	Exclusive	Non-exclusive
Rivalrous	Private Goods (Hot-dogs, Autos)	Commons (Fishing Grounds, Clean Air)
Non-Rivalrous	Club Goods (Satellite Transmission, Bridges)	Public Goods (National Defence, Justice)

1. Spillover effects usually appear in the column where there is non-exclusivity; because it is difficult to control the access to them.

2. Externalities in particular are non-exclusive; it is difficult to exclude someone from the effects of pollution or noise, etc.

3. This brings us to the Coase Theorem where we approach this problem from the question of whether private agreements (Coasian Bargaining) can resolve the externalities postulated in the absence of transaction costs.

10.3 Coase Theorem

1. Pigouvian solution to externalities is to internalize them through taxation and re-aligning the private costs to that of the social cost levels.

2. This eliminates the Harberger's Triangle Deadweight Loss from the traditional analytical perspective

3. Coase is suggesting that from the societal perspective there might be alternative ways of correcting the externalities when there are no transaction costs involved in bargaining (parties involved will find an efficient solution)

4. This is so whether there is property rights defined or not; because property rights merely sets the initial 'endowment' and assigns the legal claim that each party has.

10.3.1 Without Property Rights

1. Assuming no assignment of rights associated with the externality, we have a case where Party A does $20 (face-value) worth of damage to Party B;

   (a) Suppose Party A can pay $10 to prevent the damage and Party B could incur $5 to correct the damage.

   (b) Pigouvian taxes would charge Party A with $20 but Party A will simply spend $10 to avoid the tax; but clearly this is inefficient.

   (c) Under Coasian bargaining, Party A could even pay $5 to B or B could simply expend $5 to correct the damage - society only bears $5 ultimately, the most efficient outcome.

2. There's however, transaction costs involved because without property rights there is no means of establishing who should be shouldering the burden of the damage; this can of course be internalized through integration but it's not always practical.

10.3.2 With Property Rights

1. Property rights establishes the legal claims and ensures that there is a default arrangement on which party actually bears the burden.

2. Nevertheless, the assignment of property rights influences ultimate transaction costs borne by the society due to difficulties in collective action - also a Public goods problem.